Neutrino Astrophysics

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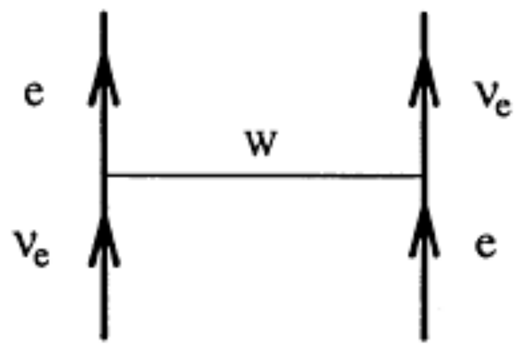
Coherent Medium-Enhanced Neutrino Flavor Transformation

Coherently propagating active neutrinos acquire effective masses (like index of refraction) via forward scattering on particles that carry weak charge.

The A potential arises from charged current forward exchange



A schematic view of neutrino effective masses

The current-current Lagrangian for neutrino-electron scattering

$$\begin{cases} L_{total} = \overline{\Psi}_\nu (i\not\partial - m_\nu)\Psi_\nu + \overline{\Psi}_e (i\not\partial - m_e)\Psi_e \\ -\dfrac{G_F}{\sqrt{2}}(\overline{\Psi}_\nu \gamma^\mu (1-\gamma_5)\Psi_\nu)(\overline{\Psi}_e \gamma_\mu (1-\gamma_5)\Psi_e) \end{cases}$$

From this we can define a potential stemming from the electron background:

$$A^\mu \equiv \frac{G_F}{\sqrt{2}}\left[\overline{\Psi}_e \gamma^\mu (1-\gamma_5)\Psi_e\right] = (\varphi, \mathbf{A})$$

The neutrino Lagrangian is then:

$$L_\nu = \overline{\Psi}_\nu \left(i\not\partial - \not A(1-\gamma_5) - m_\nu\right)\Psi_\nu$$

The equation of motion (Dirac equation) corresponding to this is . . .

$$\underbrace{\left(i\frac{\partial}{\partial t} - \varphi\right)\Psi_\nu = \left[\boldsymbol{\alpha} \bullet (\tfrac{1}{i}\nabla - \mathbf{A}) + \beta m_\nu\right]\Psi_\nu}$$

and the dispersion relation is . . .

$$\left(E_\nu - \varphi\right)^2 = \left(\mathbf{p}_\nu - \mathbf{A}\right)^2 + m_\nu^2$$

$$E_\nu^2 = \mathbf{p}_\nu^2 + \left[m_\nu^2 + 2E_\nu\varphi - 2\mathbf{p}_\nu \bullet \mathbf{A} + \left(\mathbf{A}^2 - \varphi^2\right)\right]$$

$$2E_\nu \frac{G_F}{\sqrt{2}}\left(\overline{\Psi}_e\gamma^0(1-\gamma_5)\Psi_e\right) \approx 1.5\times 10^{-7}\,\mathrm{eV}^2\left(\frac{\rho N_a Y_e}{1\mathrm{g\ cm}^{-3}}\right)\left(\frac{E_\nu}{\mathrm{MeV}}\right)$$

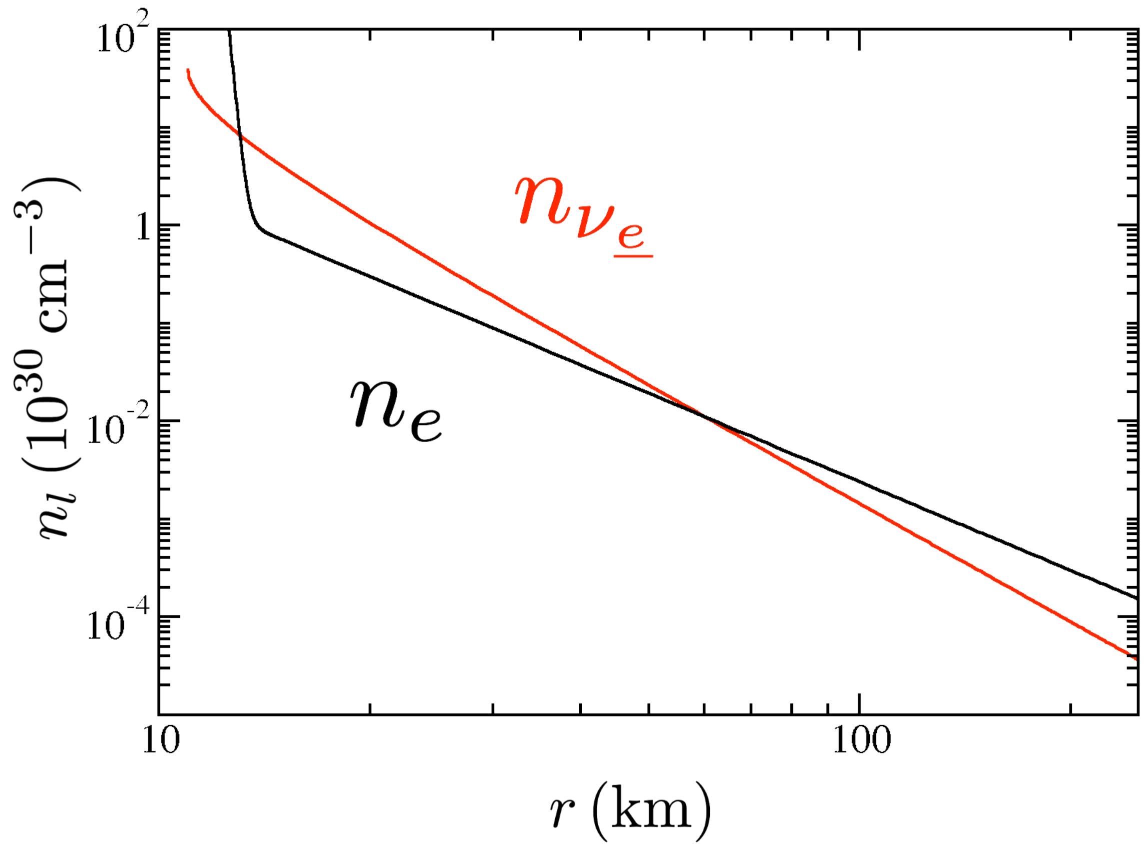
$$E_\nu^2 = \mathbf{p}_\nu^2 + \left[m_\nu^2 + 2E_\nu\langle\varphi\rangle - 2\langle\mathbf{p}_\nu \bullet \mathbf{A}\rangle + \left(\langle\mathbf{A}^2\rangle - \langle\varphi^2\rangle\right)\right]$$

$$\sim G_F^2$$

$$2\langle p_\nu A\cos\theta\rangle \approx 2E_\nu\langle\varphi\rangle\langle\cos\theta\rangle$$

zero if electron distribution is isotropic

Neutrino-neutrino forward scattering
can dominate the potential in the early universe and
supernovae



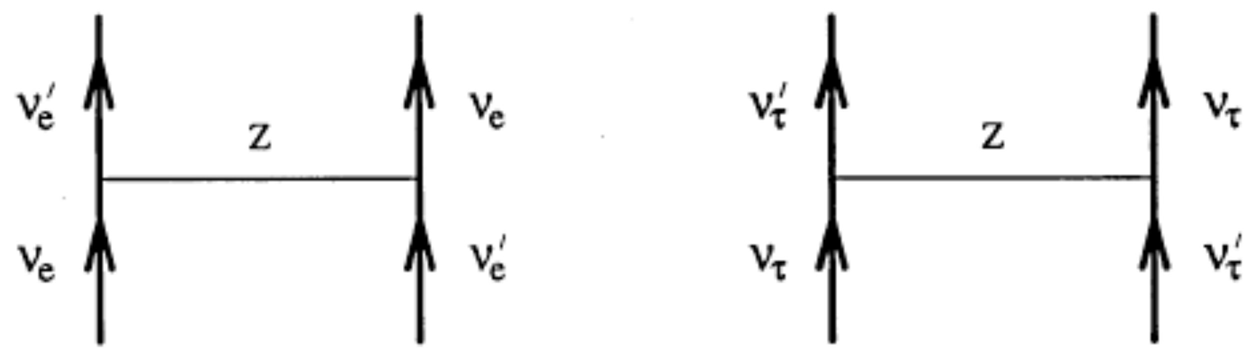
Late-time

Hot-bubble

S_b=140 k_B

L_ν=10^{51} erg/s

The neutrino “background” potentials arise from neutral current forward exchange scattering, e.g.,



flavor diagonal potential **B**

flavor off-diagonal potential $\mathbf{B}_{e\tau}$

Low-Temperature Neutrino Forward Scattering Potentials

$$H(\nu_s) \approx 0$$

$$H(\nu_e) = \sqrt{2}G_F\left(n_e - \tfrac{1}{2}n_n\right) + \sqrt{2}G_F\left[2\left(n_{\nu_e} - n_{\bar{\nu}_e}\right) + \left(n_{\nu_\mu} - n_{\bar{\nu}_\mu}\right) + \left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)\right]$$

$$H(\nu_\mu) = \sqrt{2}G_F\left(-\tfrac{1}{2}n_n\right) + \sqrt{2}G_F\left[\left(n_{\nu_e} - n_{\bar{\nu}_e}\right) + 2\left(n_{\nu_\mu} - n_{\bar{\nu}_\mu}\right) + \left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)\right]$$

$$H(\nu_\tau) = \sqrt{2}G_F\left(-\tfrac{1}{2}n_n\right) + \sqrt{2}G_F\left[\left(n_{\nu_e} - n_{\bar{\nu}_e}\right) + \left(n_{\nu_\mu} - n_{\bar{\nu}_\mu}\right) + 2\left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)\right]$$

Flavor Basis Evolution $|\Psi_{\nu_\alpha}\rangle$ neutrino born as ν_α (α=e,τ) at neutrino sphere

$$\Psi_f \equiv \begin{bmatrix} a_{e\alpha}(t) \\ a_{\tau\alpha}(t) \end{bmatrix} \quad \begin{cases} a_{e\alpha}(t) \equiv \langle \nu_e | \Psi_{\nu_\alpha}(t) \rangle \\ a_{\tau\alpha}(t) \equiv \langle \nu_\tau | \Psi_{\nu_\alpha}(t) \rangle \end{cases}$$

$$\Delta \equiv \frac{\delta m^2}{2E_\nu}$$

$$i\frac{\partial \Psi_f}{\partial t} \approx \left[\left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{A}{2} + \alpha_\nu \right) \hat{I} + \frac{1}{2} \begin{pmatrix} A + B - \Delta \cos 2\theta & \Delta \sin 2\theta + B_{e\tau} \\ \Delta \sin 2\theta + B_{\tau e} & \Delta \cos 2\theta - A - B \end{pmatrix} \right] \Psi_f$$

Potentials

$$\begin{cases} A = \sqrt{2} G_{\mathrm{F}} \left(n_{e^-} - n_{e^+} \right) \\ B = \sqrt{2} G_{\mathrm{F}} \int (1 - \cos\theta_{\mathbf{pq}}) \left(\left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t) \right]_{ee} - \left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t) \right]_{\tau\tau} \right) d^3\mathbf{q} \\ B_{e\tau} = 2\sqrt{2} G_{\mathrm{F}} \int (1 - \cos\theta_{\mathbf{pq}}) \left[\hat{\rho}_{\mathbf{q}}(t) - \hat{\bar{\rho}}_{\mathbf{q}}(t) \right]_{e\tau} d^3\mathbf{q} \end{cases}$$

Density Operators

$$\hat{\rho}_{\mathbf{p}}(t)\, d^3\mathbf{p} \equiv \sum_\alpha dn_{\nu_\alpha} |\Psi_{\nu_\alpha}(t)\rangle\langle\Psi_{\nu_\alpha}(t)|$$

$$\hat{\bar{\rho}}_{\mathbf{p}}(t)\, d^3\mathbf{p} \equiv \sum_\alpha dn_{\bar{\nu}_\alpha} |\Psi_{\bar{\nu}_\alpha}(t)\rangle\langle\Psi_{\bar{\nu}_\alpha}(t)|$$

e.g., number of neutrinos of alpha flavor in a pencil of directions and energy

$$dn_{\nu_\alpha} \approx \frac{L_{\nu_\alpha}}{\pi R_\nu^2} \frac{1}{\langle E_{\nu_\alpha} \rangle} \left(\frac{d\Omega_\nu}{4\pi} \right) f_{\nu_\alpha}(E_\nu)\, dE_\nu$$

First let's discuss the case with small flavor off-diagonal potential - this will give us a physical feel for how Mikheyev-Smirnov-Wolfenstein (MSW) medium-enhanced neutrino flavor transformation works.

Example: There is no flavor off-diagonal potential in the active-sterile neutrino flavor mixing channel.

Consider active-active neutrino mixing:

in vacuum

$$\left|\nu_\alpha\right\rangle = \cos\theta\left|\nu_1\right\rangle + \sin\theta\left|\nu_2\right\rangle$$

$$\left|\nu_\beta\right\rangle = -\sin\theta\left|\nu_1\right\rangle + \cos\theta\left|\nu_2\right\rangle$$

here α,β = e,μ,τ

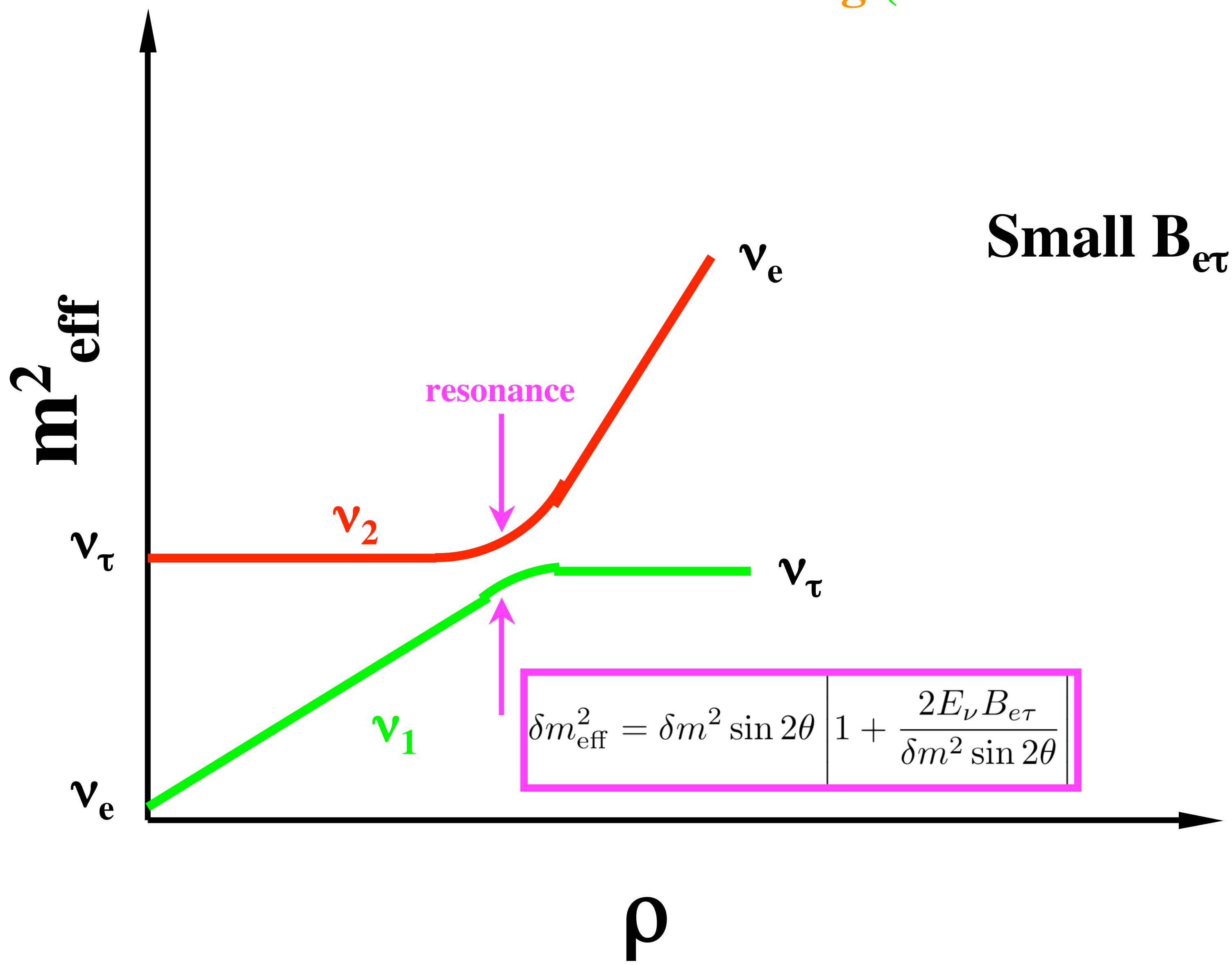
in "medium," in the early universe or supernova

$$\left|\nu_\alpha\right\rangle = \cos\theta_M(t)\left|\nu_1(t)\right\rangle + \sin\theta_M(t)\left|\nu_2(t)\right\rangle$$

$$\left|\nu_\beta\right\rangle = -\sin\theta_M(t)\left|\nu_1(t)\right\rangle + \cos\theta_M(t)\left|\nu_2(t)\right\rangle$$

See for example:
Abazajian, Fuller, Patel, Phys. Rev. D64, 023501 (2001).

Neutrino Mass Level Crossing (MSW Resonance)

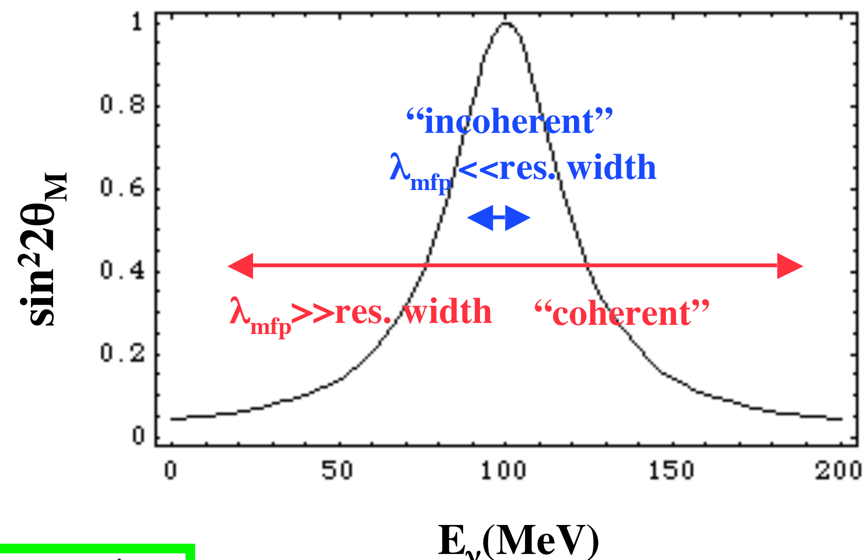


ordinary MSW evolution of neutrino flavors

MSW resonance at neutrino energy

$$E_\nu = \frac{\delta m^2 \cos 2\theta}{2(A+B)} \approx (0.02\ \text{MeV})\left(\frac{\delta m^2 \cos 2\theta}{3\times 10^{-3}\ \text{eV}^2}\right)\left(\frac{10^6\ \text{g cm}^{-3}}{\rho(\text{Y}_\text{e}+\text{Y}_\nu)}\right)$$

At a given location expect only neutrinos in a narrow energy range to experience efficient flavor conversion while anti-neutrino conversion is suppressed. With the small measured Neutrino mass-squared differences we expect significant flavor conversion only at low densities.



time/position - dependent mixing angle and mass-states

$$|\nu_e\rangle = \cos\theta_M(t)|\nu_1(t)\rangle + \sin\theta_M(t)|\nu_2(t)\rangle$$

$$|\nu_\tau\rangle = -\sin\theta_M(t)|\nu_1(t)\rangle + \cos\theta_M(t)|\nu_2(t)\rangle$$

effective $in-medium$ mixing angles θ_M and $\bar{\theta}_M$

$$|\nu_e\rangle = \cos\theta_M(t)|\nu_1(t)\rangle + \sin\theta_M(t)|\nu_2(t)\rangle$$
$$|\nu_\tau\rangle = -\sin\theta_M(t)|\nu_1(t)\rangle + \cos\theta_M(t)|\nu_2(t)\rangle$$

$$|\bar{\nu}_e\rangle = \cos\bar{\theta}_M(t)|\bar{\nu}_1(t)\rangle + \sin\bar{\theta}_M(t)|\bar{\nu}_2(t)\rangle$$
$$|\bar{\nu}_\tau\rangle = -\sin\bar{\theta}_M(t)|\bar{\nu}_1(t)\rangle + \cos\bar{\theta}_M(t)|\bar{\nu}_2(t)\rangle$$

MSW : $\theta_M = \pi/4$ at resonances only; $\bar{\theta}_M \approx 0$

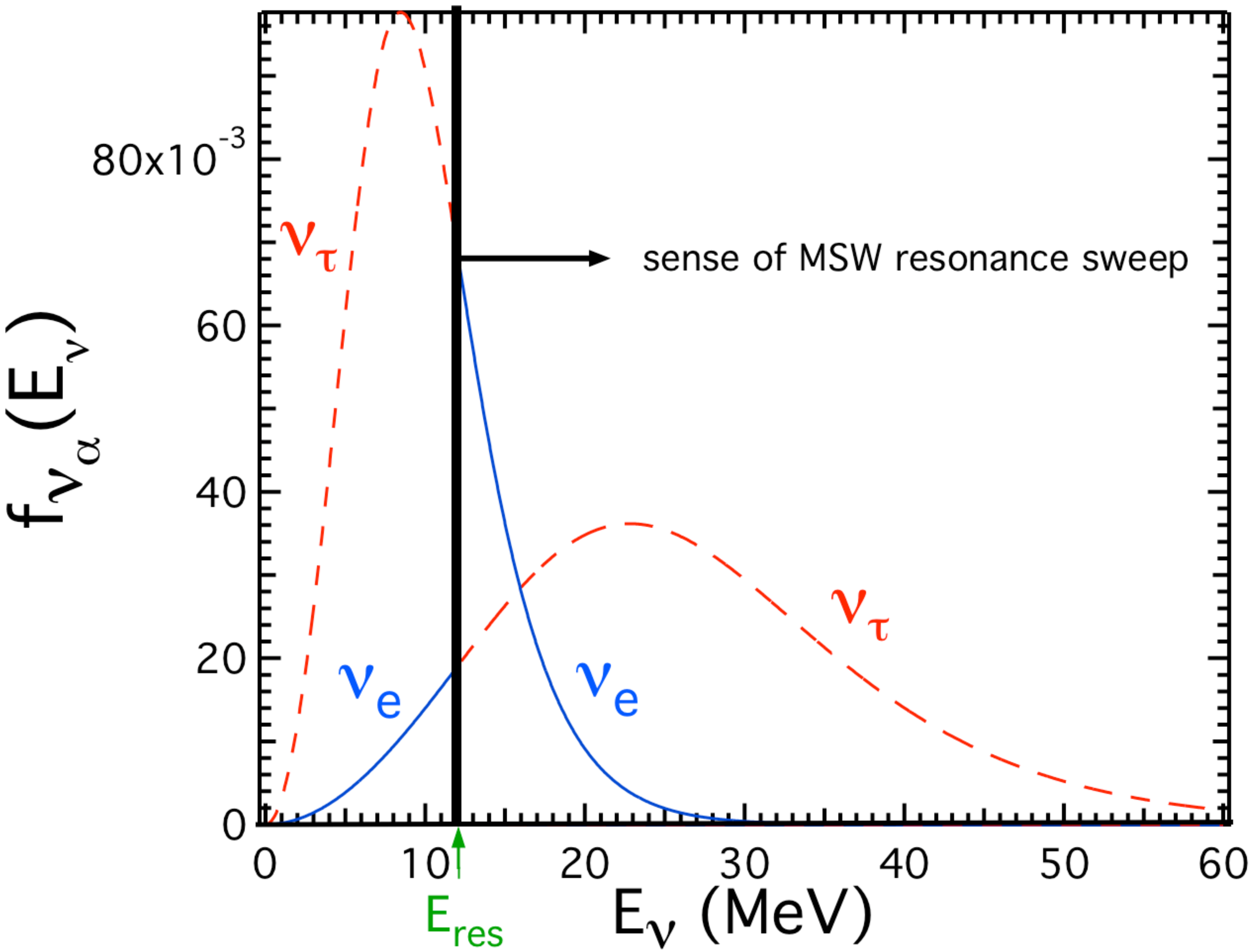
Probability that neutrino changes its flavor on propagating through an MSW resonance:

$$P \approx 1 - P_{\mathrm{LZ}}$$

Where Landau-Zener "jump probability"
(the probability that we jump between mass tracks)
is

$$P_{\mathrm{LZ}} = e^{-\frac{\pi}{2}\gamma}$$

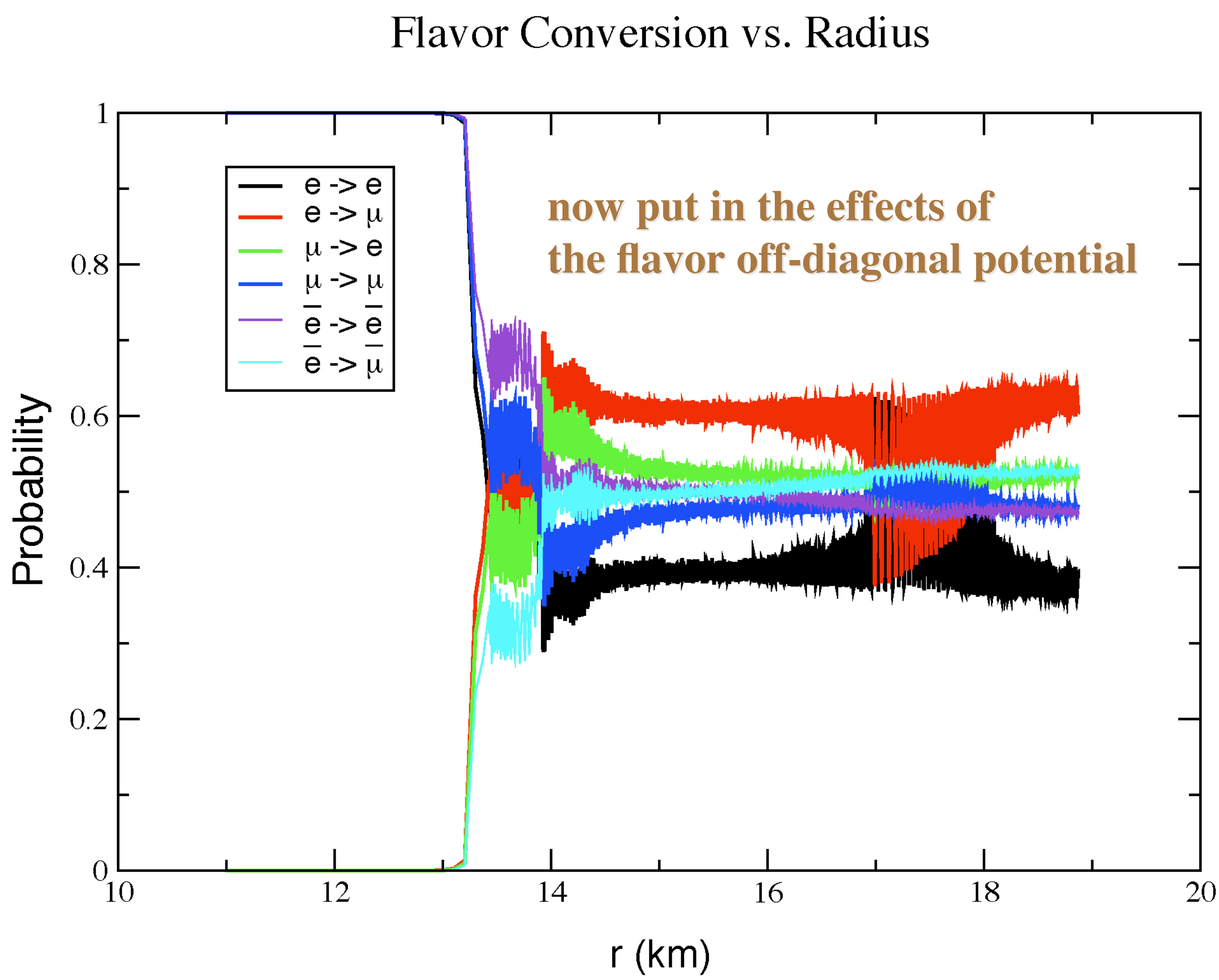
$$\text{adiabaticity parameter} \quad \gamma \equiv 2\pi \left(\frac{\text{resonance width}}{\text{oscillation length at res.}} \right)$$



$f_{\nu_\alpha}(E_\nu)$
80x10^{-3}
60
40
20
0
ν_τ
ν_e
ν_e
ν_τ
sense of MSW resonance sweep
0
10
20
30
40
50
60
E_{res}
E_ν (MeV)

But what happens when we put in the flavor basis off-diagonal potential ?

J. Carlson (2004)



instantaneous transformation between in-medium mass states and flavor states

$$\left|\nu_e\right\rangle = \cos\theta_{\mathrm{M}}\left(t\right)\left|\nu_1\left(t\right)\right\rangle + e^{-i\delta(t)}\sin\theta_{\mathrm{M}}\left(t\right)\left|\nu_2\left(t\right)\right\rangle$$

$$\left|\nu_\tau\right\rangle = -e^{i\delta(t)}\sin\theta_{\mathrm{M}}\left(t\right)\left|\nu_1\left(t\right)\right\rangle + \cos\theta_{\mathrm{M}}\left(t\right)\left|\nu_2\left(t\right)\right\rangle$$

$$\Delta_{\mathrm{eff}}\cos 2\theta_{\mathrm{M}} = \Delta\cos 2\theta - A - B$$

$$\Delta_{\mathrm{eff}}\, e^{i\delta}\sin 2\theta_{\mathrm{M}} = \Delta\sin 2\theta + B_{e\tau}$$

$$\Delta_{\mathrm{eff}} = \sqrt{\left(\Delta\cos 2\theta - A - B\right)^2 + \left|\Delta\sin 2\theta + B_{e\tau}\right|^2}$$

Fuller & Qian astro-ph/0505240

Background Dominant Solution:

$$|B_{e\tau}| \gg |A+B|$$

$$\left.\begin{aligned} \cos 2\theta_M &\to 0 \\ \sin 2\theta_M &\to 1 \\ \cos 2\bar{\theta}_M &\to 0 \\ \sin 2\bar{\theta}_M &\to -1 \end{aligned}\right\} \quad \begin{aligned} \theta_M &\to \frac{\pi}{4} \\ \bar{\theta}_M &\to \frac{3\pi}{4} \end{aligned}$$

$$\text{for real, positive} \quad B_{e\tau}$$

Large Off-Diagonal Potentials Increase Adiabaticity

…by decreasing neutrino oscillation length at resonance

$$\boxed{L_{\mathrm{osc}}^{\mathrm{res}} = \frac{4\pi E_\nu}{\delta m_{\mathrm{eff}}^2} \approx \frac{4\pi E_\nu}{\delta m^2 \sin 2\theta} \left| 1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta} \right|^{-1}}$$

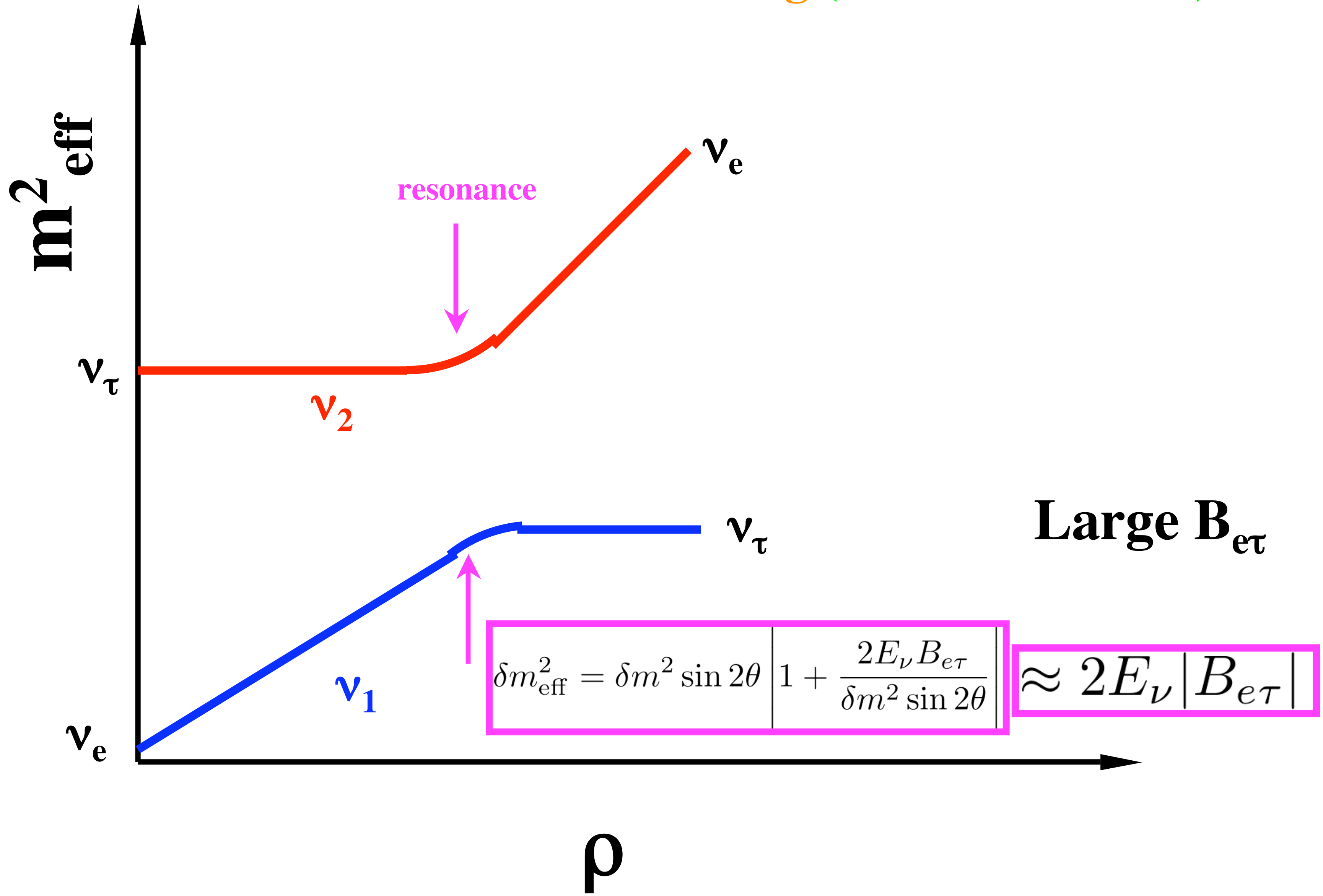
…and by increasing the resonance width $\boxed{\Delta \equiv \delta m^2 / 2E_\nu}$

$$\boxed{\delta r = \frac{dr}{dV}\,\delta V = \underbrace{\left| \frac{1}{V}\frac{dV}{dr} \right|^{-1}}_{\text{Density scale height}} \Delta \sin 2\theta \left| 1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta} \right|}$$

Adiabaticity parameter **(adiabatic if $\gamma \gg 1$)**

$$\boxed{\gamma = \frac{\delta r}{L_{\mathrm{osc}}^{\mathrm{res}}} \approx \frac{1}{2}\frac{\delta m^2 \mathcal{H}}{E_\nu}\frac{\sin^2 2\theta}{\cos 2\theta} \left| 1 + \frac{2E_\nu B_{e\tau}}{\delta m^2 \sin 2\theta} \right|^2}$$

Neutrino Mass Level Crossing (MSW Resonance)



Proto Neutron Star Envelope Density Run

(1) Hydrostatic equilibrium:

$$TS \approx \frac{G\, M_{\rm NS}\, m_p}{r} \qquad\qquad r_6 \approx \frac{22.5}{T_9\ S_{100}}$$

(2) Isentropic (constant entropy) flow & entropy in relativistic particles

$$S \approx \frac{2\pi^2}{45} g_s \frac{T^3}{n_b}$$ **constant**

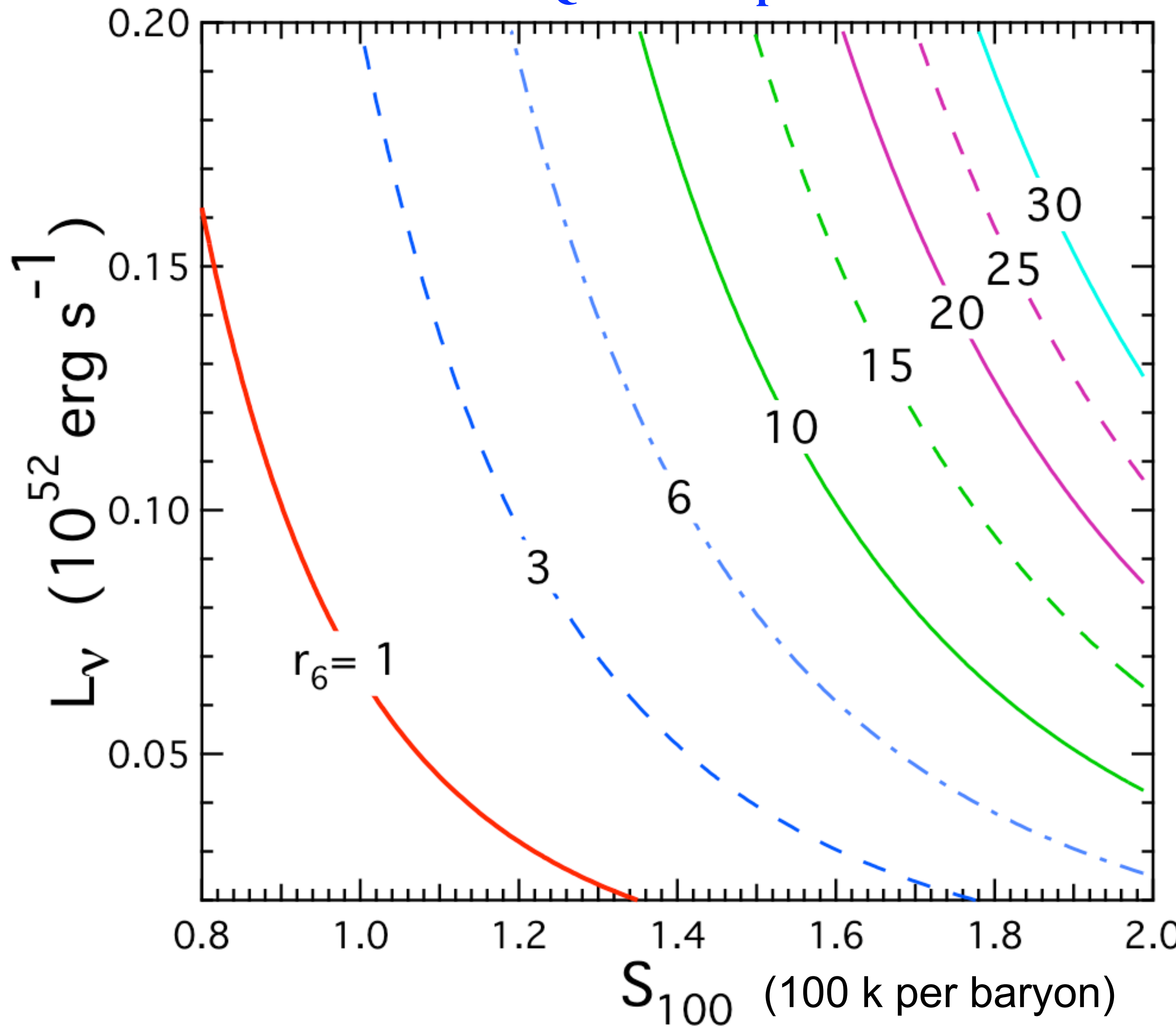
(1) + (2) ⇒

$$\boxed{n_b \approx \frac{2\pi^2}{45} g_s \left(\frac{M_{\rm NS}\ m_p}{m_{\rm pl}^2} \right)^3 S^{-4} r^{-3}}$$

Neutrino-Driven Wind, *r*-Process Regime

conditions necessary for simultaneous neutrino & antineutrino flavor conversion

Fuller & Qian astro-ph/0505240



$$\nu_e \rightleftharpoons \nu_{\mu,\tau}$$

$$\bar{\nu}_e \rightleftharpoons \bar{\nu}_{\mu,\tau}$$

$$\mathrm{r}_6 \equiv \frac{\mathrm{r}}{10^6\,\mathrm{cm}}$$

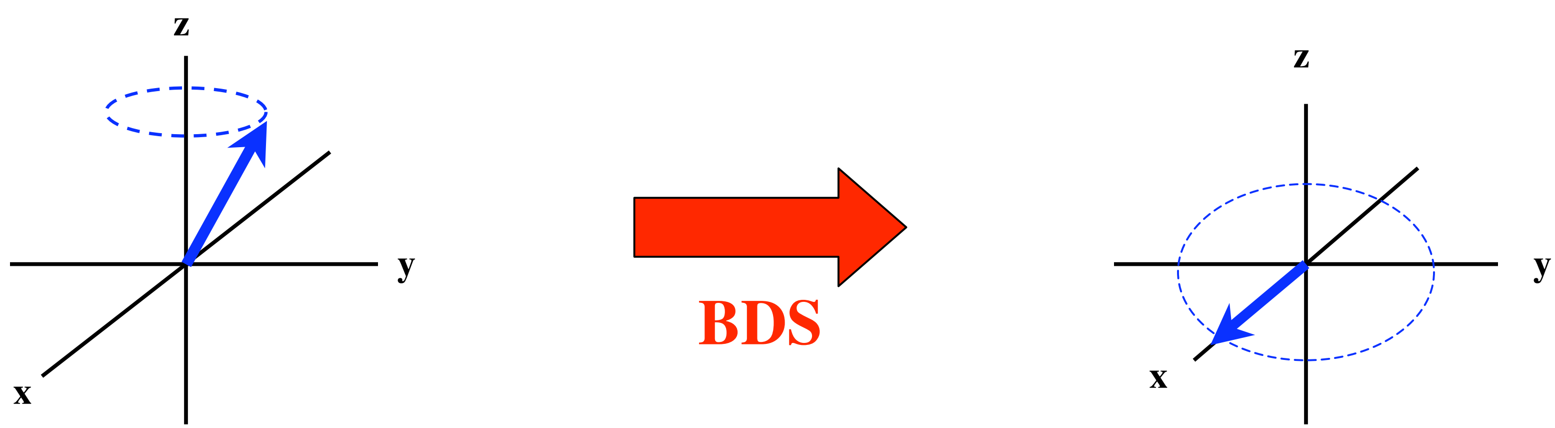
Here $\delta m^2 = 3 \times 10^{-3}\,\mathrm{eV}^2$

Spin Polarization Analogy: Spin-One representation of SU(2)

Write flavor basis density operator in terms of Pauli spin matrices:

$$\begin{pmatrix} \rho_{ee} & \rho_{e\tau} \\ \rho_{\tau e} & \rho_{\tau\tau} \end{pmatrix} = \frac{P_0 \hat{I} + P_x\sigma_x + P_y\sigma_y + P_z\sigma_z}{2} = \frac{1}{2}\begin{pmatrix} P_z + P_0 & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix}$$

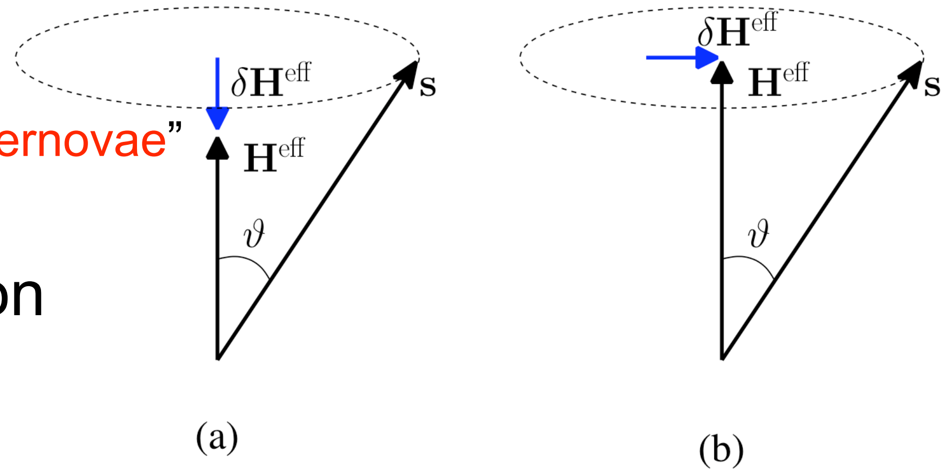
Polarization vector $\mathbf{P} \Rightarrow \{P_x, P_y, P_z\}$ **and** P_0



precession in xy-plane

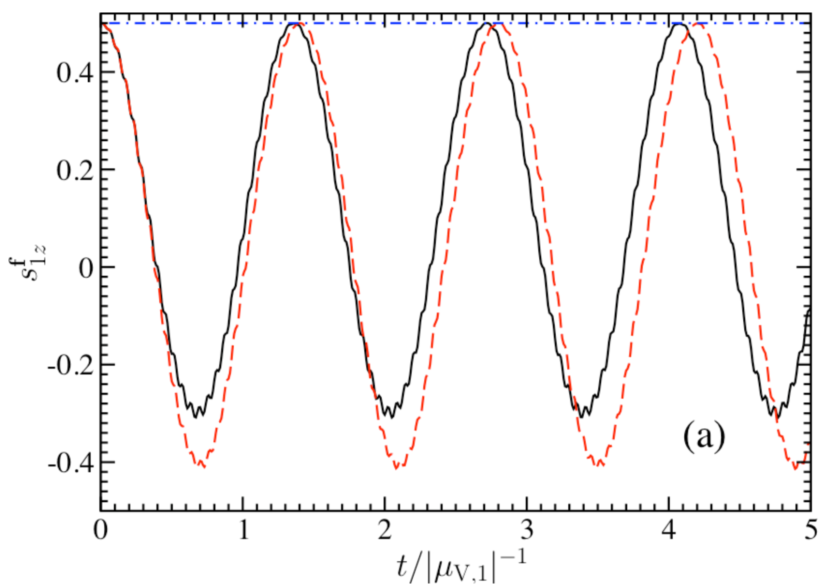
Duan, Fuller, Qian, astro-ph/0511275

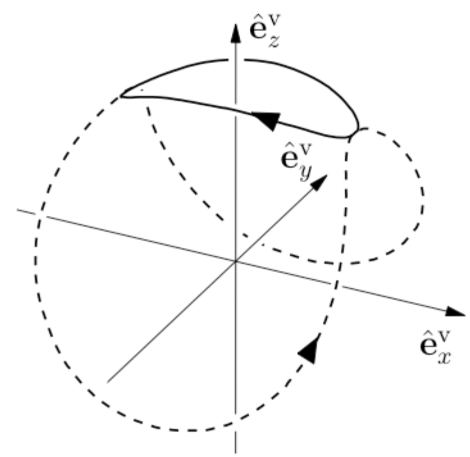
"Collective Neutrino Transformation in Supernovae"



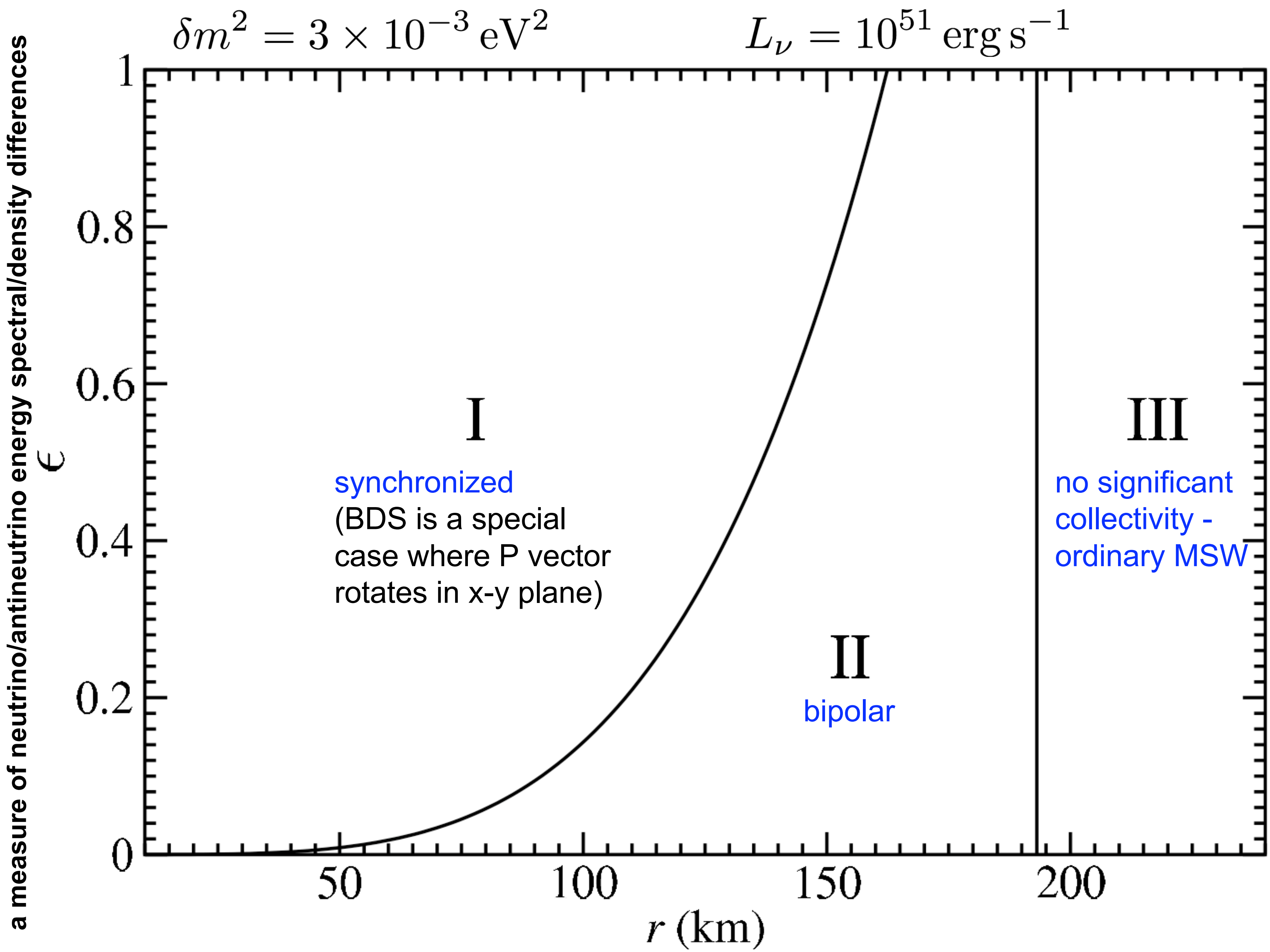
natural "fixed points" in system

Analyze neutrino flavor evolution in the "co-rotating frame," in analogy to the way electron spins are handled in, e.g., atomic clocks (see Baym "Lectures on Quantum Mechanics")





collective mode orbits

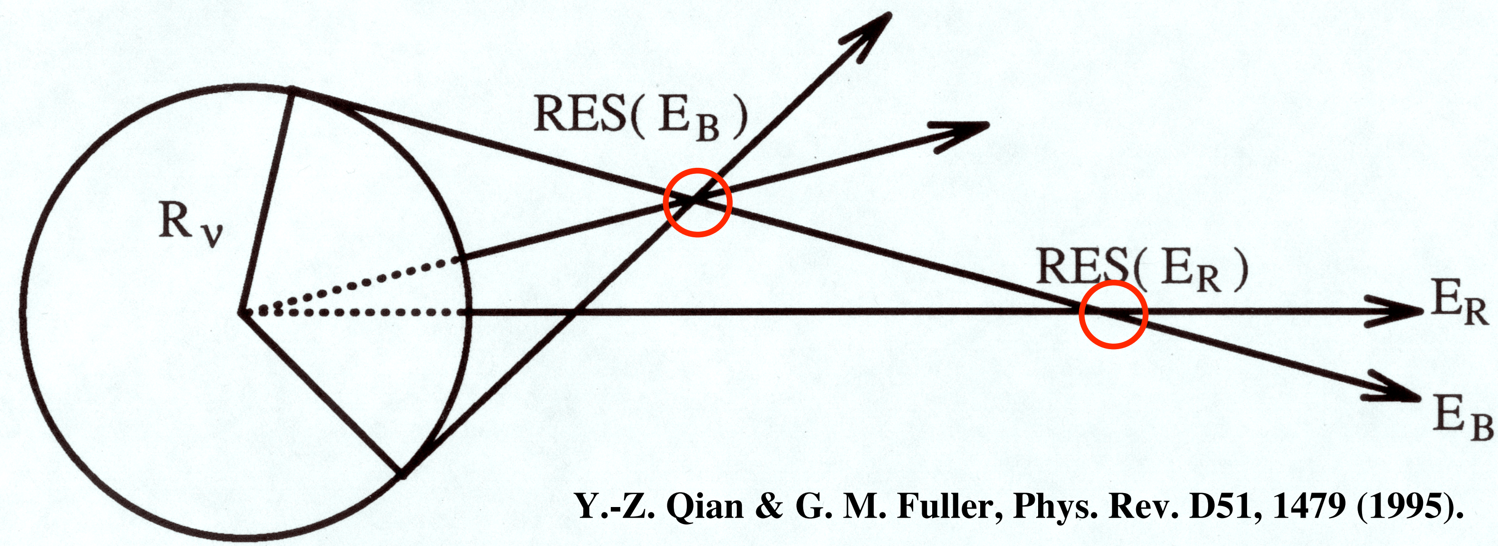


Duan, Fuller, Qian, astro-ph/0511275

So now the question is:

Does nature ever find these solutions?

The flavor amplitude evolution history of a given neutrino depends on the prior amplitude evolution histories of the background neutrinos which intersect its world line.



Y.-Z. Qian & G. M. Fuller, Phys. Rev. D51, 1479 (1995).

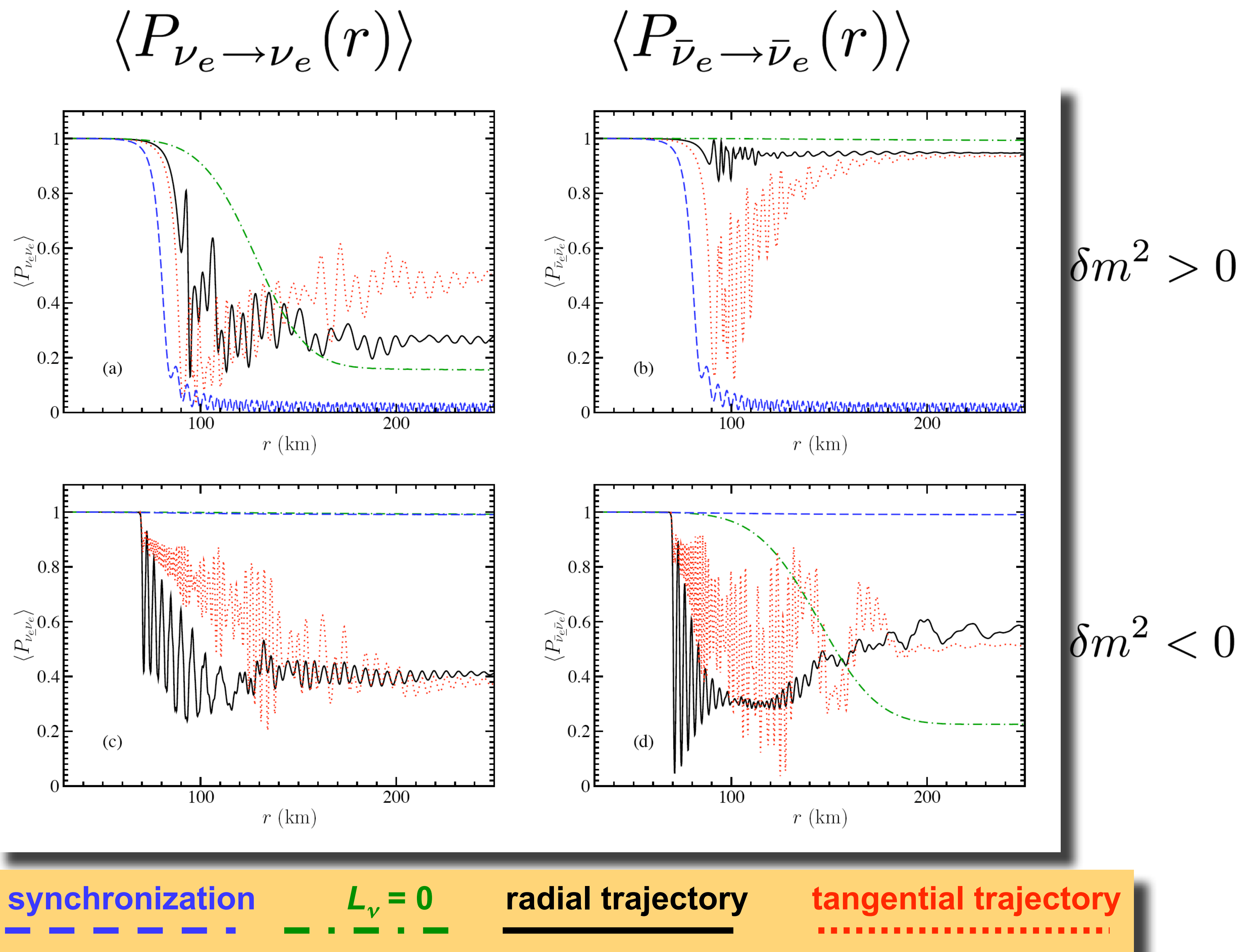
Macroscopic Quantum Coherence:
geometric and quantum entangling of flavor histories of neutrinos on intersecting trajectories.

Solve this in a mean field context

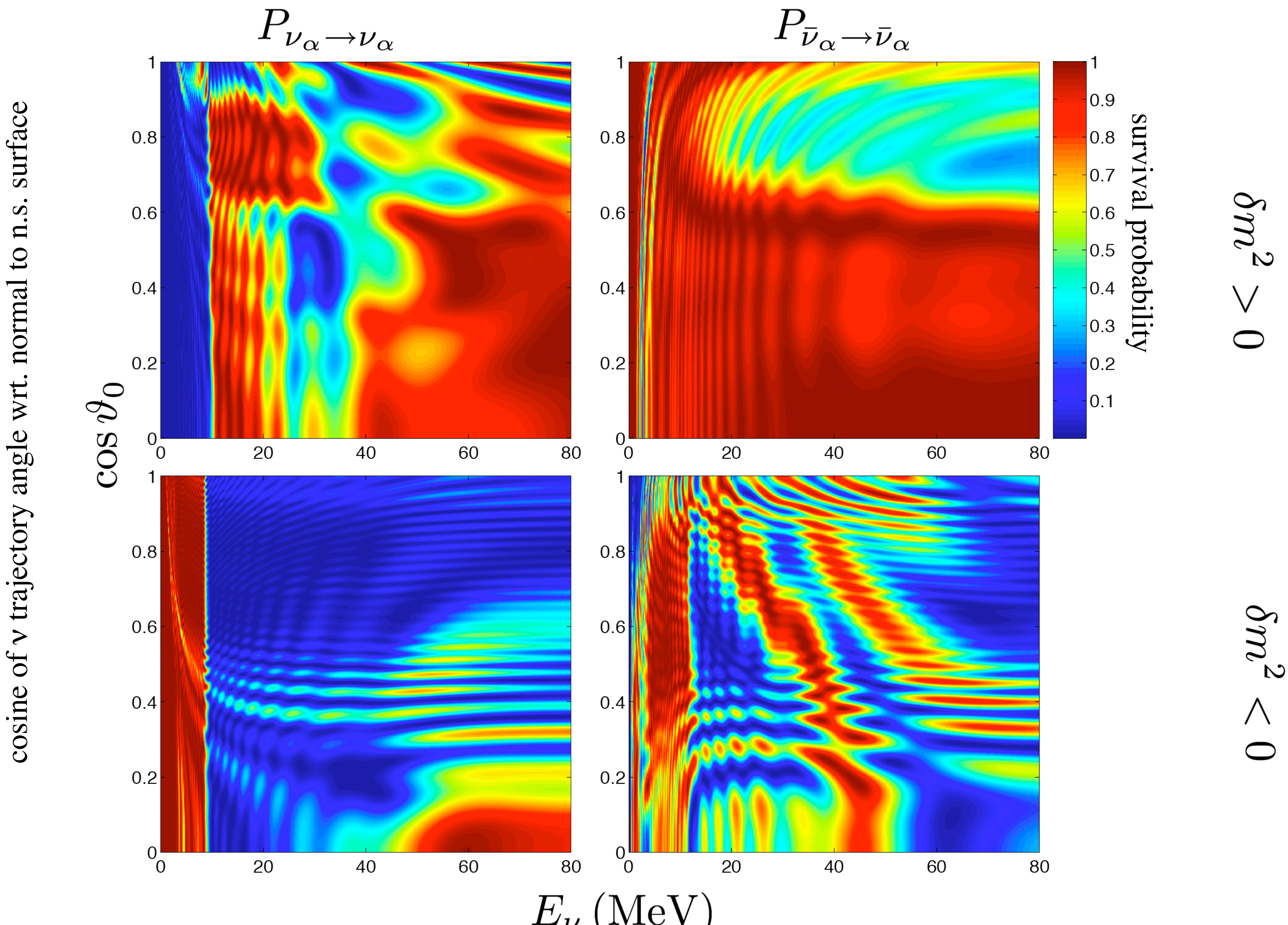
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\psi_\nu = (H_{\mathrm{vac}} + H_e + H_{\nu\nu})\psi_\nu$$

$$H_{\nu\nu} = \sqrt{2}G_{\mathrm{F}} \int (1 - \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}')(\rho_{\mathbf{q}'} - \bar{\rho}_{\mathbf{q}'})\, \mathrm{d}\mathbf{q}'$$

Neutrinos with different initial flavors, energies, and directions have correlated evolution histories!

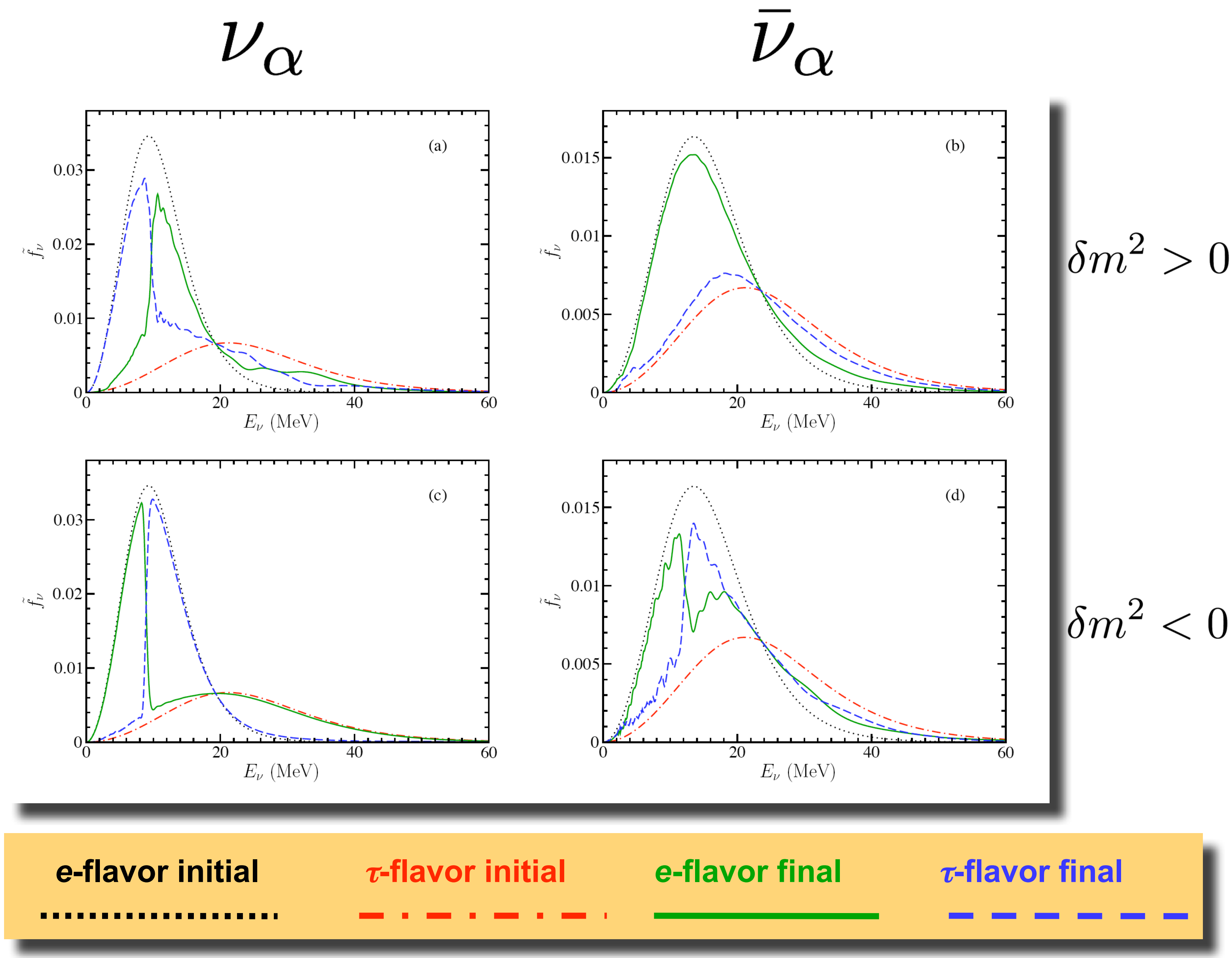


H. Duan, G. M. Fuller, J. Carlson, Y.-Z. Qian, astro-ph/0606616



macroscopic quantum coherence in neutrino/antineutrino fields

H. Duan, G. M. Fuller, J. Carlson, Y.-Z. Qian, astro-ph/0606616



H. Duan, G. M. Fuller, J. Carlson, Y.-Z. Qian, astro-ph/0606616

So, what happens when the active neutrinos transform among themselves ?

Shock re-heating *may* be enhanced, neutrino nucleosynthesis and signal affected.

(depending on where the transformation happens and on the neutrino energy spectra)

R-Process/Alpha-Effect problems may or may not get worse.

Neutrino Flavor Mixing

(active-sterile)

A possible neutrino physics solution to the alpha effect problem:

Matter-enhanced active-sterile transformation in the 3+1 scheme coupled to hydrodynamics and weak rates (feedback):

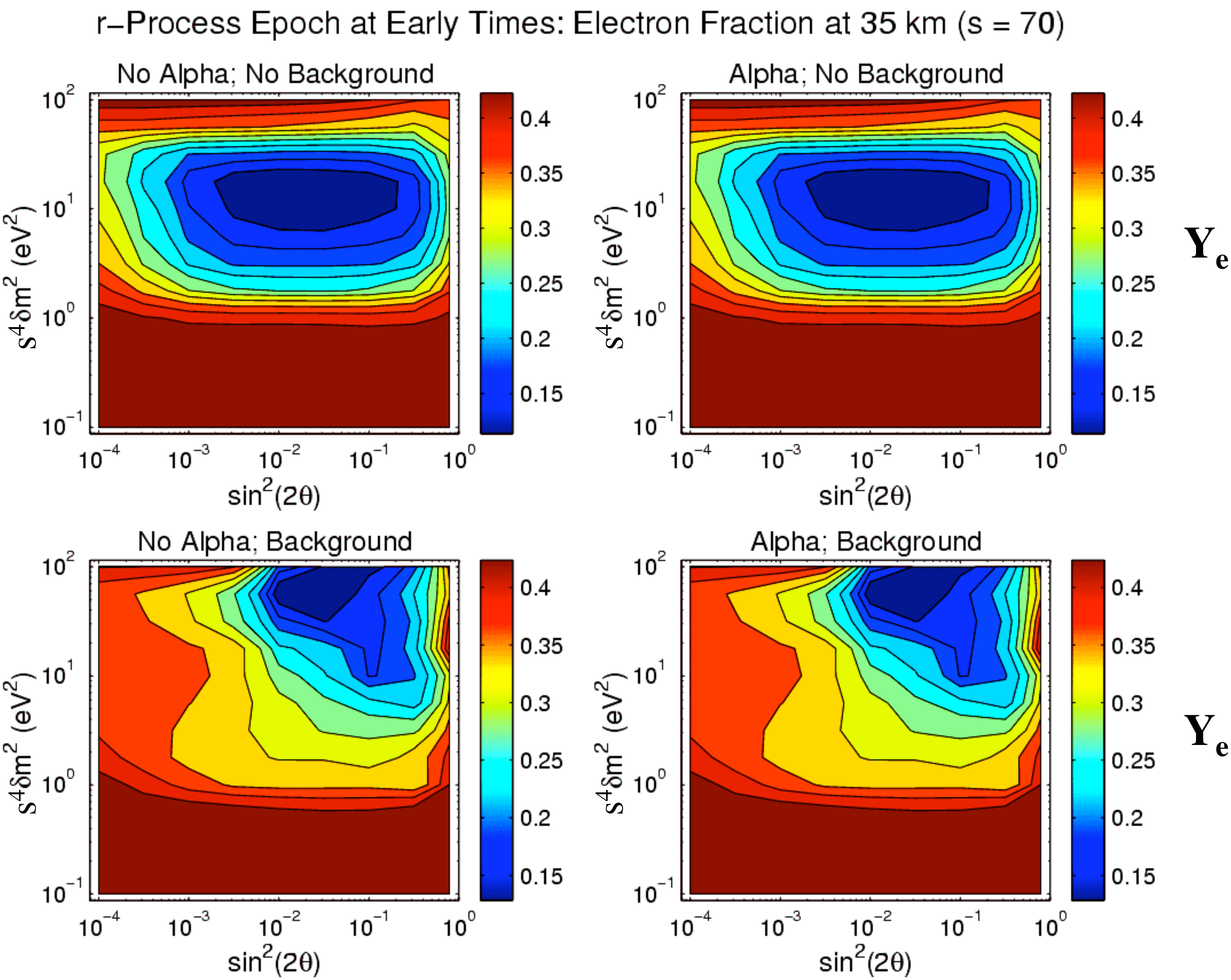
$$\nu_e \leftrightarrow \nu_s \quad \text{and} \quad \overline{\nu_e} \leftrightarrow \overline{\nu_s}$$

$$A \propto \left(Y_e - \frac{1}{3} \right)$$

⇨ **no alpha effect, extreme neutron excess, fission cycling in r-process nicely tie together abundances of 130 and 195 peaks (a fundamental feature of the observations).**

McLaughlin, Fetter, Balantekin, Fuller, Phys. Rev. C39, 2873 (1999).
Fetter, Mclaughlin, Balantekin, Fuller PRD (2002).

Patel (unpublished, 2001) has considered neutrino background effects.
See also Caldwell, Fuller, & Qian, Phys. Rev. D61, 123005 (2000) for similar “2+2” scheme.


r−Process Epoch at Early Times: Electron Fraction at 35 km (s = 70)
No Alpha; No Background
Alpha; No Background
No Alpha; Background
Alpha; Background
$s^4\delta m^2$ (eV^2)
$\sin^2(2\theta)$
Y_e
0.4
0.35
0.3
0.25
0.2
0.15